Boston Public Library
Mathematics and Maps
Title: Similarity and Dilation

Essential Question: What characteristics make objects the same shape?

Overview:
A map is a proportional, shrunken down version of an actual location. By connecting the mathematical ideas of similarity and dilation to maps, students see the mathematics in action in the real world.
In this lesson, the concept of similarity is expanded to dilation, the idea of shrinking and enlarging a shape as a transformation on the coordinate plane. Students review the properties of similarity and apply them to investigate maps of the same place but at different scales. From there, students apply the definition of dilation to experiment with similarity – this time, looking at the change in coordinates rather than the change in side lengths.

Grade Range: Grade 7-12 **Time Allocation**: 45 – 60 minutes

Objectives:
1. Students will define dilation and explain the definition.
2. Students will understand the relationship between similarity and dilation.
3. Students will provide evidence that two shapes are dilations.

Common Core Curriculum Standards:

Grade 7 – Geometry - Congruence and similarity
1. Verify experimentally that a dilation with scale factor k preserves lines and angle measure, but takes a line segment of length L to a line segment of length kL.
2. Understand the meaning of similarity: a plane figure is similar to another if the second can be obtained from the first by a similarity transformation (a rigid motion followed by a dilation).
3. Solve problems involving similar figures and scale drawings. *Include computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.*

Grade 8 – Geometry – Congruence and Similarity
Explain using similarity transformations the meaning of similarity for triangles as the equality of all pairs of angles and the proportionality of all pairs of sides.

Grades 9-12 – Geometry – Similarity

1. Understand that dilating a line produces a line parallel to the original. (In particular, lines passing through the center of the dilation remain unchanged.)
2. Understand that the dilation of a given segment is parallel to the given segment and longer or shorter in the ratio given by the scale factor. A dilation leaves a segment unchanged if and only if the scale factor is 1.
3. Use triangle similarity criteria to solve problems and to prove relationships in geometric figures.

Procedure:

Part 1: Introduction

1. Hand out the United States Railway map and the map of middle America. Have the students answer the following questions on their handout.
    1. Compare the maps. What are differences between these two maps?
    2. What is the relationship between the two maps in terms of the size of Pennsylvania?

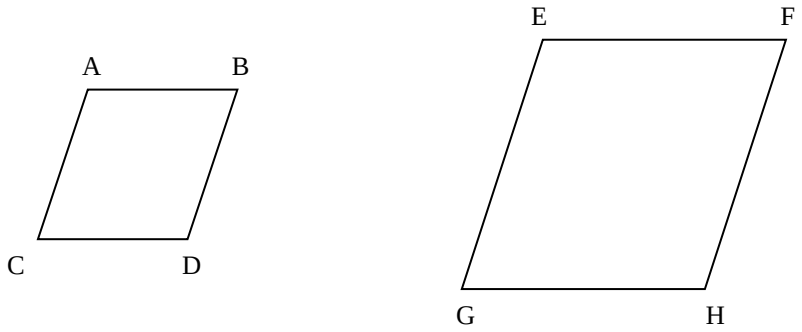
2. Discuss the student responses. Lead students to notice that the maps are of some of the same places but one is an enlarged, detailed version of the bigger map, with the actual state of Pennsylvania the biggest of all. Discuss what this idea means with the class. What does this mean specifically about Pennsylvania – in terms of its sides lengths, curves and angles?

Part 2: Similarity

1. Review the properties of similarity. Have students fill in their handout.

    Two shapes are similar if
    a. Corresponding angles are congruent.
    b. Corresponding sides are proportional.

2. Have students take notes while you review the meaning of the two properties with the following example.



Parallelograms ABCD and EFGH are similar if the following are true:

Corresponding angles are congruent:	Corresponding sides are proportional:
$\angle A \cong \angle E$ $\angle B \cong \angle F$ $\angle C \cong \angle G$ $\angle D \cong \angle H$	$\frac{AB}{EF} = \frac{AC}{EG} = \frac{CD}{GF} = \frac{BD}{FH}$

3. Have students outline the state of Pennsylvania in the two maps. What would they need to do to get some evidence that the two versions of Pennsylvania are similar? Have students check three angles and three side lengths to verify that the two states of Pennsylvania meet the properties of similarity. They should complete the chart in their handout.

Part 3: Dilations

1. The idea of similarity relates to another idea, dilation. Give students the definition of dilation as described below and explain the notation.

A dilation is a mathematical transformation that preserves the shape of an object but may change the size.

A dilation of a point in a coordinate plane can be found by multiplying the x and y-coordinates of a point by the same number, n.

$$D(x, y) = (nx, ny)$$

The number n is called the scale factor of the transformation.

Similar shapes are dilations.

If we can show evidence that our maps of Pennsylvania are dilations of each other, then we have proven that they are similar.

2. To show that they are dilations, we need to put the Pennsylvania maps on an x-y coordinate plane. On the graph paper provided, have students draw an x and y-axis. They will only need quadrant one at this time.

3. Have students cut out the state of Pennsylvania on the two maps. They should also cut out the state of New York at the same time. This will be used later in the lesson.

4. After the cutting, students should have two versions of Pennsylvania – a small version and a large version. Using the smallest version, they should align the bottom left corner of the map with the origin (0, 0) and mark the major corners of the shape on the graph paper. They can connect the points to get the rough shape of Pennsylvania – it won't be very accurate in terms of the shape of Pennsylvania because not all the points need to be marked.

5. Repeat the process for the larger version of Pennsylvania. Use a different color for each set of points.

6. For all the points drawn, label them with their coordinates. Have students estimate the when the coordinates fall between two whole numbers.

7. Have students complete the chart describing patterns they see in the coordinate points.

8. Discuss with students: Are these maps of Pennsylvania dilations? If so, what is the scale factor?

9. Have students find one more point on the smallest map of Pennsylvania and use the scale factor to find the coordinates of that location on the medium and larger sized maps.

Part 4: Property of Dilations

1. Have students examine the two dilations of the state of Pennsylvania. What observations can they make about the line segments of the smaller version compared to the line segments of the larger version? Students should notice that side lengths change by the scaling factor and corresponding sides are parallel.

2. Have students verify that their observations are true by measuring sides lengths (they should already have some of the data from section 2) and checking angles to verify the parallel relationship.

Part 6: Assessment

1. Have students complete the assignment verifying that the map of New York from the Middle States map and the New York from the United States maps are dilations.

2. Collect the assignments and review answers.

Materials Needed:

- Maps Used:
  - Map of Middle States of America (http://maps.bpl.org/id/14114)
  - United States Railway Map (http://maps.bpl.org/id/15602)

(Note: Maps are replicated at the end of the handout – the map of Massachusetts and the United States map would be easier to use if printed in landscape form instead of portrait.)

- Markers (for outlining states)
- Scissors
- Pencils
- Rulers
- Protractors

Similarity and Dilation

Name _____

Date _____

Part 1: Introduction

Review the map of the United States and the map of the Middle States of America. Then, respond to the following:

1. Compare the maps. What are differences between these two maps?
   _____
   _____
   _____

2. Find the state of Pennsylvania on each map. What is the relationship between the two maps in terms of the size of Pennsylvania?
   _____
   _____
   _____

Part 2: Similarity

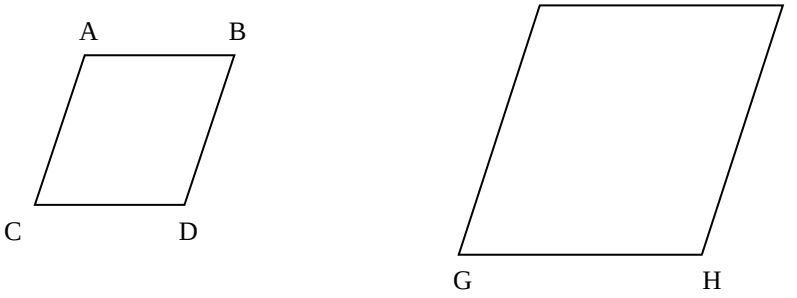
Review the properties of similarity.

Two shapes are similar if

1. _____

2. _____

For example:



Parallelograms ABCD and EFGH are similar if the following are true:

3. Carefully outline the state of Pennsylvania in each of the two maps. What evidence would you need to verify that the two versions of Pennsylvania are similar?

   _____
   _____
   _____
   _____

4. Choose three corresponding angles in the shape of Pennsylvania and compare measurements.

Angle	Angle Measurement on United States map	Angle Measurement on Middle States map

What conclusion can you draw from the measurements of corresponding angles?

_____
_____
_____

5. Complete the chart to investigate patterns in the distances in the shape of Pennsylvania.

Distance	**Distance on United States Map**	**Distance on Middle States Map**	**How much larger? Ratio of** $\frac{\text{Larger Distance}}{\text{Smaller Distance}}$

What conclusion can you draw from the measurements of corresponding sides?

_____
_____
_____

6. Based on the measurements of corresponding angles and sides, are the two different size maps of Pennsylvania similar? Explain using your data as evidence and by referring to the properties of similarity.

_____
_____
_____
_____
_____
_____

Part 3: Dilations

1. The idea of similarity relates to another idea, dilation. Give students the definition of dilation as described below and explain the notation.

A dilation is a mathematical transformation that preserves the shape of an object but may change the size.

A dilation of a point in a coordinate plane can be found by multiplying the x and y-coordinates of a point by the same number, n.

$$D(x, y) = (nx, ny)$$

The number n is called the scale factor of the transformation.

Similar shapes are dilations.

Notes:

2. To show that the two maps of Pennsylvania are dilations, we need to put the Pennsylvania maps on an x-y coordinate plane. On the graph paper provided, draw an x and y-axis. You will only need quadrant I at this time.

3. Cut out the state of Pennsylvania in each of the two maps. Note that you will also need the state of New York for each map as well – you may want to cut this out now as well.

4. Using the smaller version, align the bottom left corner of the map with the origin (0, 0) and mark the major corners of the shape on the graph paper. You may connect the points to get the rough shape of Pennsylvania.

5. Repeat the process for the larger version of Pennsylvania. Use a different color for each set of points.

6. For all the points drawn, label them with their coordinates. Have students estimate the when the coordinates fall between two whole numbers.

Location	Coordinates smaller map of Pennsylvania	Coordinates on larger map of Pennsylvania	Scaling Factor
Location	**Coordinates on smaller map of Pennsylvania**	**Coordinates on larger map of Pennsylvania**	**Scaling Factor**

7. Are these maps of Pennsylvania dilations? Why or why not? And if so, what is the scale factor?

   _____
   _____
   _____
   _____
   _____
   _____

8. Using your knowledge of dilation and scale factors, respond to the following:

   a. The coordinates of the city of Pittsburg on the Pennsylvania map from the Middle States map are approximately (1.5, 3). What coordinates would Pittsburg have on the Pennsylvania map from the United States map? Show your work or explain your answer.

b. The coordinates of the city of Philadelphia on the Pennsylvania map from the United States map are approximately (6.4, .75). What coordinates would Philadelphia have on the Pennsylvania map from the Middle States map? Show your work or explain your answer.

Part 4: Property of Dilations

1. Examine the two dilations of the state of Pennsylvania. What observations can you make about the line segments of the smaller version compared to the line segments of the larger version?

_____
_____
_____
_____
_____
_____

2. How can we verify our observations? Show your work below.

_____
_____
_____
_____
_____

Part 5: Assessment

Repeat the process from above to verify that the map of New York from the Middle States map and the map of New York from the United States maps are dilations.

Location	Coordinates on Smaller Map	Coordinates on Larger Map	Scaling Factor

Are these maps of New York dilations? Why or why not? And if so, what is the scale factor?

_____

_____

_____