Boston Public Library
Mathematics and Maps
Title: Introduction to Functions

Essential Question: What is a function and how to we apply the idea to a real world context?

Overview:
The idea of a functional relationship can make more sense to students if that relationship is grounded in a real world situation instead a purely mathematical one. By applying the definition of function to the idea of map scale, students see why the rules and language clarify the situation by giving us tools to better understand maps as well as mathematics.

Grade Range: 8 – 12 **Time Allocation**: 45 minutes

Objectives:
1. Students will describe key vocabulary and notation associated with functions.
2. Students will represent functions in terms of diagrams, tables, graphs and equations.

Common Core Curriculum Standards

Grade 8 – Functions – Function Concepts
1. Understand that a function from one set (called the domain) to another set (called the range) is a rule that assigns to each element of the domain (an input) exactly one element of the range (the corresponding output). The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. *Function notation is not required in Grade 8.*
2. Evaluate expressions that define functions, and solve equations to find the input(s) that correspond to a given output.
3. Compare properties of two functions represented in different ways (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
4. Understand that a function is linear if it can be expressed in the form $y = mx + b$ or if its graph is a straight line. *For example, the function $y = x^2$ is not a linear function because its graph contains the points (1,1), (–1,1) and (0,0), which are not on a straight line.*

Grades 9 – 12 - Functions – Interpreting Functions
1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x.

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2. Use function notation and evaluate functions for inputs in their domains.
3. Compare properties of two functions represented in different ways (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, draw conclusions about the graph of a quadratic function from its algebraic expression.*
4. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

Procedure:

Part 1: Introduction

Have students review the National Highways Map of Massachusetts. Project the map as well so students can see the scale information clearly.

1. What is the purpose of this map?
2. What is the scale? What does it represent?
3. Is map scale a function? That is if we take a distance on the map and scale it up to an actual distance, is this relationship a function?

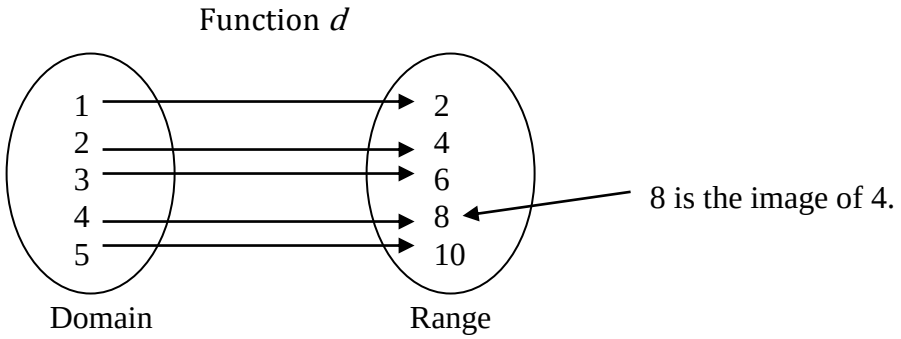
Part 2: Background on Functions

1. To answer the third questions, we need to know what a function is. Define function and key vocabulary with students.

A **function** is a rule which takes certain numbers as inputs and assigns to each input number exactly one output number. The output is a function of the input.

The inputs and outputs are called variables. The set of all inputs is called the **domain** of the function. The set of all outputs is called the **range** of a function. An individual element of the ranges is called an **image**.

2. Show students an example to clarify the meaning of the vocabulary.



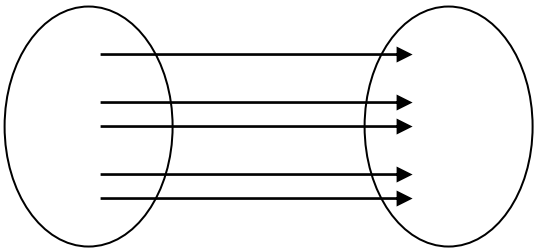
3. Introduce function notation with examples.

   $d(2) = 4$ $\qquad$ $d(5) = 10$ $\qquad$ The image of 3 is 6.

Part 3: Scale as a Function

1. Bring the students back to the discussion question. Is scale a function? What would it mean if the relationship between distances on the map and in the real world was not a function? What is the domain? What is the range? Have student diagram the different parts.

Function *D*: Relates distances on the map to distances in the real world



Domain – Distances on the map

Range – Distances in the real world

2. To understand the rule for our function, we look at the map scale. The scale is shown as a representative fraction, 1:1,000,000. This means that distances in the real world are 1,000,000 times larger than distances on the map. Thus, the rule for our function is as follows:

   $D(x) = 1{,}000{,}000 \cdot x$

   where x represents a distance on the map and *D(x)* represents the related actual distance

3. Have students practice a bit with the notation and the rule.

a. What does D(2.5) mean? Calculate it and state your answer in a sentence.
b. If $D(x) = 3{,}250{,}000$, what is x? Explain what this means in words.

Part 3: Representing Functions

1. As we've already seen, functions can be represented as a diagram and an equation. There are other ways to represent a function as well – in words, in a table, as coordinate points and as a graph.

2. Lead students through the assignment to show the different ways that a function can be represented. Be explicit about how the domain and range change from discrete to continuous for tables and coordinate points versus equations and graphs. Also keep in mind the situation: in our map scale function, could a distance be negative? Be sure to define the function for the given context as well.

Part 4: Assessment

Students should complete the assessment section of the assignment demonstrating their understanding of how to represent functions. Note that the last questions on the assignment are an extension designed to preview student understanding before a discussion of composition of functions. Teachers can collect these assignments and review both the notes section of the assignment and the responses.

Materials:

- Map - A National Highways Map of Massachusetts (http://maps.bpl.org/id/12686)
- Calculators

Introduction to Functions

Name _____

Date _____

Part 1: Introduction

Review the National Highways Map of Massachusetts.

1. What is the purpose of this map? _____
   _____
   _____

2. What is the scale? What does it represent? _____
   _____
   _____

3. Is map scale a function? If we take a distance on the map and scale it up to an actual distance, is this relationship a function?
   _____
   _____
   _____
   _____

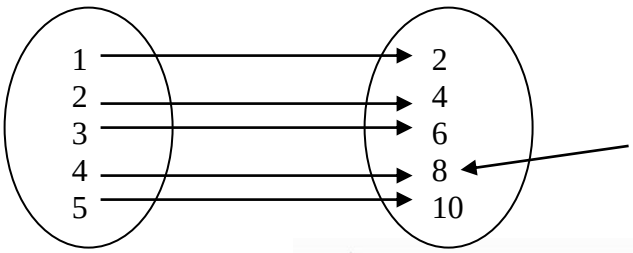
Part 2: Background on Functions

To answer the third question, we need to know what a function is. Define function and key vocabulary with students.

A **function** is a rule which takes certain numbers as inputs and assigns to each input number exactly one output number. The output is a function of the input.

The inputs and outputs are called variables. The set of all inputs is called the **domain** of the function. The set of all outputs is called the **range** of a function. An individual element of the ranges is called an **image**.

Notes:



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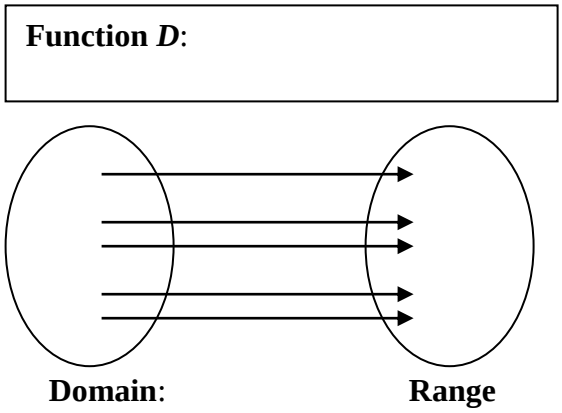
Part 3: Scale as a Function



1. The scale on the National Highways map is shown as a representative fraction. Write it below.

   This means that _____.

   Rule for Function: _____

   where *x* represents _____ and *D(x)* represents _____.

2. Given our rule,

   a. What does D(2.5) mean in words? _____
   _____

   b. Calculate D(2.5). Show your work.

c. If $D(x) = 3{,}250{,}000$, what is the value of x? Explain what you are finding in words.

_____

_____

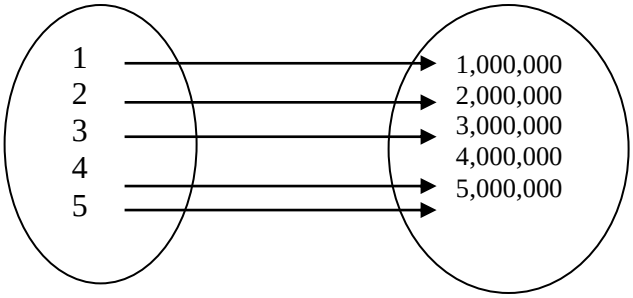
d. Determine the value of x if $D(x) = 3{,}250{,}000$. Show your work.

Part 4: Representing Functions

Functions can be represented in a variety of ways. Using the Map Scale function from our National Highway Map of Massachusetts, we can show the different representations.

1. **Diagram**

Function D



Domain: _____ Range: _____

Image of 3: _____ Notation: _____

2. **Table**

Function D

x	y
1	
2	
3	
4	
5	
6	

Domain: _____ Range: _____

Image of 6: _____ Notation: _____

3. **Coordinate Points**

{(1,1,000,000), (2, _____), (3,3,000,000), (4,4,000,000), (5,_____), (6,6,000,000)}

Domain: _____ Range: _____

Image of 1: _____ Notation: _____

4. **Words**

The function *D(x)* _____
_____
_____
_____

Domain: _____

Range: _____

Image of 23: _____ Notation: _____

5. **Equation**

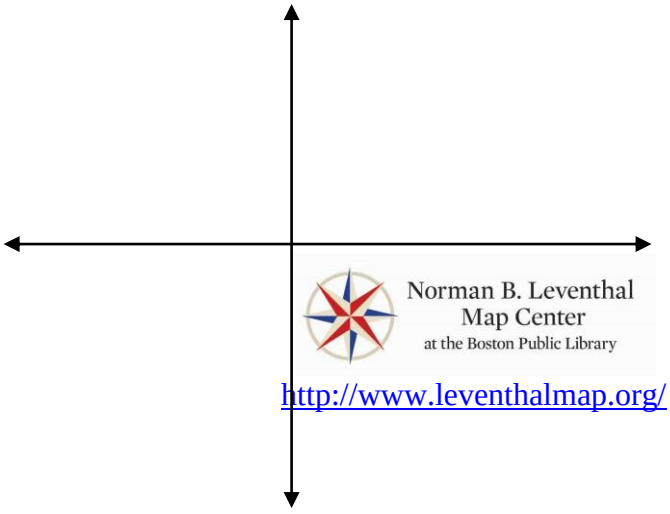
D(x) = _____

Domain: _____

Range: _____

Image of 23: _____ Notation: _____

6. **Graph**



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$D(x) =$ _____

Domain: _____

Range: _____

Image of 100: _____ Notation: _____

Part 5: Assessment

Define the relation f as follows:

f takes elements in the domain, adds 4, then maps them to elements in the range

Why is this relation a function?

_____

_____

_____

Represent the function f in the different ways listed below.

Diagram	**Table**	**Coordinate Points**
Equation	**Graph**	

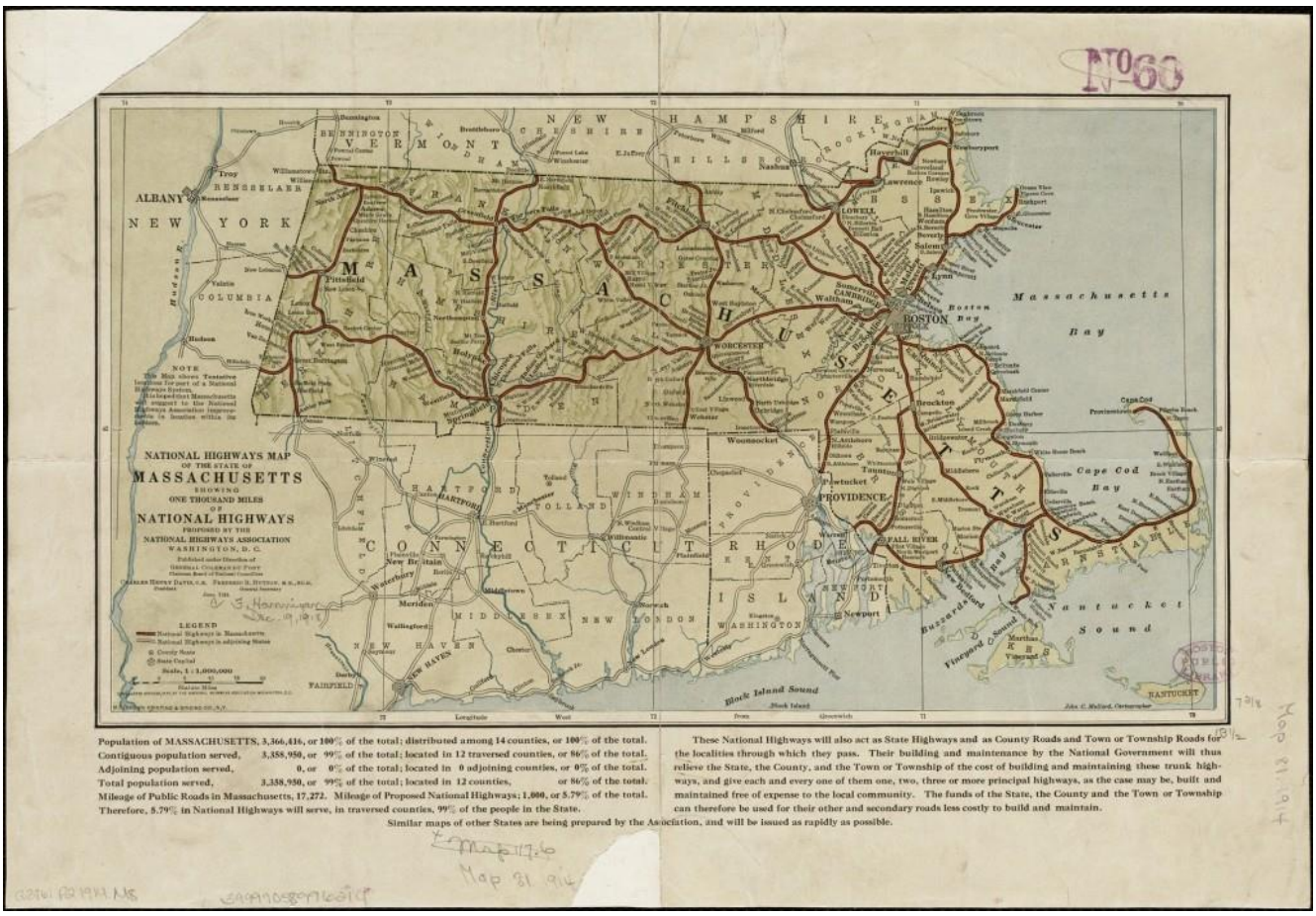
What is the domain of function f? _____

What is the range of function f? _____

Find the following values:

$f(3) =$ _____ $f(0) =$ _____ $f(-8) =$ _____

$f(x+2) =$ _____ $f(3x) =$ _____ $f(2x+1) =$ _____



Population of MASSACHUSETTS, 3,366,416, or 100% of the total; distributed among 14 counties, or 100% of the total.
Contiguous population served, 3,358,950, or 99% of the total; located in 12 traversed counties, or 86% of the total.
Adjoining population served, 0, or 0% of the total; located in 0 adjoining counties, or 0% of the total.
Total population served, 3,358,950, or 99% of the total; located in 12 counties, or 86% of the total.
Mileage of Public Roads in Massachusetts, 17,272. Mileage of Proposed National Highways; 1,000, or 5.79% of the total.
Therefore, 5.79% in National Highways will serve, in traversed counties, 99% of the people in the State.

These National Highways will also act as State Highways and as County Roads and Town or Township Roads for the localities through which they pass. Their building and maintenance by the National Government will thus relieve the State, the County, and the Town or Township of the cost of building and maintaining these trunk highways, and give each and every one of them one, two, three or more principal highways, as the case may be, built and maintained free of expense to the local community. The funds of the State, the County and the Town or Township can therefore be used for their other and secondary roads less costly to build and maintain.

Similar maps of other States are being prepared by the Association, and will be issued as rapidly as possible.

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