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MATHEMATICS AND MAPS
TITLE: AREA AND GEOMETRY**



Essential Question: How do we find the area and perimeter of common shapes?

Overview:

Maps lend themselves to exploring the concepts of perimeter and area because they deal with the idea of distance and living space. To build their understanding, students translate distance measurements to their corresponding square unit and use these units to calculate perimeters and areas of shapes. The lesson extends the “counting units” method to investigating patterns in the areas of rectangles, introducing and applying the formulas.

Grade Range: Grade 3 - 6

Time Allocation: 20 – 40 minutes per lesson

Objectives:

1. Students will define perimeter and area and relate them to the real-world context of maps.
2. Students will determine the perimeter of a shape using repeated addition.
3. Students will determine the area of a shape using square units.
4. Students will develop the formula for area and perimeter of a rectangle and apply them to solve problems.

Common Core Curriculum Standards:

Grade 3 Standards - Measurement and Data: Perimeter and area

1. Understand and use concepts of area measurement.
 - a. A square with side length 1 unit, called a unit square is said to have one square unit of area, and can be used to measure area.
 - b. A plane figure which can be covered without gaps or overlaps by n unit squares has an area of n square units. Areas of some other figures can be measured by using fractions of unit squares or using figures whose areas have been found by decomposing other figures.
 - c. When measuring an area, if a smaller unit of measurement is used, more units must be iterated to measure the area in those units.
 - d. Determine and compare areas by counting square units. *Use cm^2 , m^2 , in^2 , ft^2 , and improvised units.*
2. Understand that multiplication of whole numbers can be represented by area models; a rectangular region that is a length units by b length units (where a and b are whole numbers) and tiled with unit squares illustrates why the rectangle encloses an area of $a \times b$ square units.
3. Solve problems involving perimeters of polygons.
 - a. Add given side lengths, and multiply for the case of equal side lengths.

- b. Find an unknown length of a side in a polygon given the perimeter and all other side lengths; represent these problems with equations involving a letter for the unknown quantity.
- c. Exhibit rectangles with the same perimeter and different area, and with the same area and different perimeter.

Grade 3 – Geometry: Structuring rectangular shapes

1. Understand that rectangular regions can be tiled with squares in rows and columns, or decomposed into such arrays.
2. Structure a rectangular region spatially by decomposing it into rows and columns of squares. Determine the number of squares in the region using that spatial structure (e.g., by multiplication or skip counting).

Grade 4 - Measurement and Data: Perimeter and Area

1. Understand that if a region is decomposed into several disjointed pieces, then the area of the region can be found by adding the areas of the pieces (when these areas are expressed in the same units).
2. Apply the formulas for area of squares and rectangles. Measure and compute whole-square-unit areas of objects and regions enclosed by geometric figures which can be decomposed into rectangles. *Limit to situations requiring products of one-or two-digit numbers.*

Grade 5 - Measurement and Data: Units of Measure

1. Understand that quantities expressed in like units can be added or subtracted giving a sum or difference with the same unit; different quantities may be multiplied to obtain a new kind of quantity (e.g., as when two lengths are multiplied to compute an area, or when an area and a length are multiplied to compute a volume).
2. Understand that when measuring a quantity, if a smaller unit is used, more units must be iterated to measure the quantity in those units.

Grade 6 – Geometry: Properties of Area, Surface Area and Volume

1. Understand that plane figures can be decomposed, reassembled, and completed into new figures; use this technique to derive area formulas.
2. Solve problems involving area, volume and surface area of objects.

Procedure:

Lesson 1: Introduction

1. Hand out the Massachusetts Highway Map (<http://maps.bpl.org/id/12686>) and show it on the overhead. Have students answer the following questions as a class:
 - a. What states are shown in this map?
 - b. Outline Massachusetts, Connecticut and Rhode Island. Which one is bigger? How do you know? (Note that “bigger” is subjective – students could look at area or perimeter to determine size.)

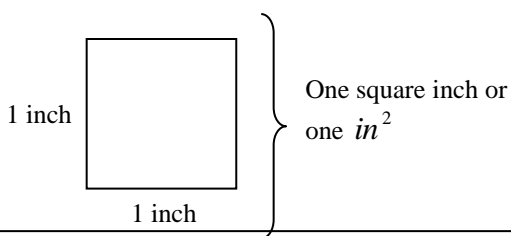
- Clarify two types of measurements that can be used to determine which is bigger – perimeter and area. Define each term both mathematically and in terms of a map; perimeter is the distance around a state while area is the amount of space inside the perimeter. Use the classroom as an example: Perimeter relates to coloring around the space on the floor while area relates to coloring the entire floor.
- Continue the discussion – why is it important to know the perimeter of a place on a map? Why is it important to know the area?

Lesson 2: Measuring Perimeters

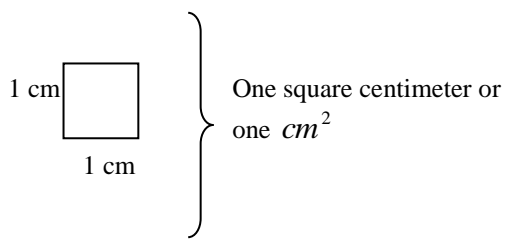
- To compare perimeters, we will first need a unit of measure. Using a ruler and scissors, measure and cut out 15 pieces of yarn that measure one inch. Model how to place the pieces of yarn along the edge of Massachusetts without overlapping to find and approximation for the perimeter. If needed, discuss $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$ of an inch lengths or round off to the nearest whole inch.
- Have pairs of students use the one-inch pieces of yarn to approximate the perimeters of Connecticut and Rhode Island and record the information on their handout. Have students generalize the coastlines, matching up the yarn to the general shape of the state.
- What happens if we change the unit? Using a ruler and scissors, measure and cut out 20 pieces of a different color of yarn that measure 1 centimeter. Model how to place the pieces of yarn along the edge of Massachusetts without overlapping to approximate the perimeter, once again generalizing the length of the coastline.
- Have the partners use the one-centimeter pieces of yarn to approximate the perimeters of Connecticut and Rhode Island and record the information.
- Discuss with students their observations – what happens to the perimeter when the unit is changed to one that is smaller?

Lesson 3: Measuring Areas

- To measure the amount of space within Massachusetts, we will use area. Hold up the inch-long pieces of yarn. Show that it doesn't make sense to find how many of those fill up the space – the pieces of yarn are a measure of length not a measure of area. To use the yarn to create an area, we need to build a square. A square with one inch sides is called a "square inch" or " in^2 ".



2. Pass out square-inch grids and have students cut out the squares. Have students color the squares using the same color as the 1-inch yarn lengths. Model how to lay the squares within Massachusetts to find an approximation for the area in square inches. Lead the discussion to include “partial” blocks; $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$ ’s of a square inch or round off to the nearest square inch.
3. Have students use the one-inch squares to approximate the areas of Connecticut and Rhode Island and record the information.
4. What happens if we change the unit? Suppose our squares have a side length of 1 centimeter instead of 1 inch? This is called a square centimeter or one cm^2 .



5. Pass out centimeter grids and have students cut out the squares. Have students color the squares using the same color as the 1-centimeter yarn lengths. Model how to lay the squares within Massachusetts to find an approximation for the area in square centimeters.
6. Have students use the one-centimeter squares to approximate the areas of Connecticut and Rhode Island and record the information.
7. Discuss with students their observations – what happens to the area when the unit is changed to one that is smaller?

Lesson 4: Patterns in Rectangles

Have students explore the area and perimeter of rectangles and generalize the pattern into a formula.

1. Using their 1-inch square pieces, have students construct rectangles with a variety of dimensions and find both the areas and perimeters. They should work with a partner to check each other’s work. Be sure that they use the correct units.
2. Discuss the patterns that they see. From this, have students state the rules for a rectangle:

$$\text{Area of a Rectangle} = \text{Length} \times \text{Width}$$

$$\text{Perimeter of a Rectangle} = 2 \times \text{Length} + 2 \times \text{Width}$$

3. Have students continue to work with their partner. Give students the following challenges:
 - a. Create two rectangles with an area of 12 square units but with different perimeters.
 - b. Create two rectangles with perimeter of 10 units but with different areas.
 - c. A rectangle has a perimeter of 18 and one side length of 6. What is the other side length?
4. Give students another copy of the Massachusetts Highway map. Instead of finding perimeter using yarn lengths and area using individual square inches, have students draw a rectangle to approximate the area of Rhode Island. Model how to draw a rectangle, measure the side lengths in inches and determine an approximation for the area and perimeter.
5. Have students repeat the process for Connecticut and complete their handout with side length measurements as well as perimeter and area calculations. They should work with the same partner to check each other's work.
6. Massachusetts is a little trickier because it consists of multiple rectangles. Adding the areas of these rectangles gives an approximation for the area but adding the perimeters does not because of the overlapping sides. Go through the process with students to find the area. Discuss the issue with finding an estimate for the total perimeter of Massachusetts using the same method. Determine the approximate perimeter of Massachusetts with students indicating which sides are on the "outside" to be included in the total.

Lesson 5: Practice

Have students complete the practice assignment with area and perimeter extension problems.

Lesson 6: Assessment

1. Have students complete the assignment on finding perimeter and area using the United States map (<http://maps.bpl.org/id/15602>).
2. Collect the assignments and review answers.

Materials Needed:

- Maps Used:
 - Massachusetts Highway Map (<http://maps.bpl.org/id/12686>)
 - Map of United States (<http://maps.bpl.org/id/12602>)

(Note: Maps are replicated at the end of the handout – they would be easier to use if printed in landscape form instead of portrait.)

- Yarn
- Square inch and centimeter grid sheets
- Markers (for outlining states)
- Scissors
- Pencils



Area and Perimeter

Name _____

Date _____

Lesson 1: Introduction

Vocabulary	Definition/Notes	Units
Area		
Perimeter		



Area and Perimeter

Name _____

Date _____

Lesson 2: Measuring Perimeters

1. Using a ruler and scissors, measure and cut out pieces of yarn that measure one inch. Draw out one inch below.

One Inch Length:

2. Using the one-inch yarn lengths, follow the model of your teacher to complete the table below.

State	Number of One-Inch Yarn Lengths AROUND the State	Perimeter (Don't forget your units!)
Massachusetts		
Connecticut		
Rhode Island		

3. What happens to the perimeter when we change the unit? Using a ruler and scissors, measure and cut out pieces of yarn that measure one centimeter. Draw out one centimeter below.

One Centimeter Length:

4. Using the one-centimeter yarn lengths, follow the model of your teacher to complete the table below.

State	Number of One-Centimeter Yarn Lengths AROUND the State	Perimeter (Don't forget your units!)
Massachusetts		
Connecticut		

Rhode Island		
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5. What happens to the perimeter when the unit is changed to one that is smaller?



Area and Perimeter

Name _____

Date _____

Lesson 3: Measuring Areas

1. To measure the amount of space within Massachusetts, we will use area. Follow along with your teacher and use a ruler to draw one square inch below.

One Square Inch or One in^2 :

2. Using the one square inch squares, follow the model of your teacher to complete the table below.

State	Number of One square inch squares that fit INSIDE the state	Area (Don't forget your units!)
Massachusetts		
Connecticut		
Rhode Island		

3. What happens to the area when we change the unit? Follow along with your teacher and use a ruler to draw one square centimeter below.

One Square Centimeter or One cm^2 :

4. Using the one square centimeter squares, follow the model of your teacher to complete the table below.

State	Number of One square centimeter squares that fit INSIDE the state	Area (Don't forget your units!)
Massachusetts		
Connecticut		
Rhode Island		

5. What happens to the area when the unit is changed to one that is smaller?



Area and Perimeter

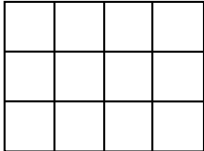
Name _____

Date _____

Lesson 4: Patterns in Rectangles

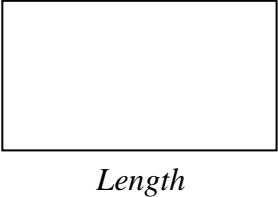
By exploring patterns in the area and perimeter of rectangles, we can find rules to help us find the answers.

Using your 1-square inch square pieces, work with your partner to construct rectangles and complete the table below. Be sure to check your units. An example has been completed for you.

Width	Length	Sketch of Solution	Perimeter	Area
3 inches	4 inches		$4 + 4 + 3 + 3 =$ <i>14 inches</i>	12 in^2
2 inches	3 inches			
5 inches	2 inches			

1. What patterns do you notice in the table above?

2. Rules for Rectangles:

Rectangle Diagram	Perimeter Rule	Area Rule
<div style="display: flex; align-items: center; justify-content: center;"> <div style="margin-right: 10px;"><i>Width</i></div>  </div>		

3. Practice the rule with the highway map of Massachusetts. This time, instead of finding the perimeter using yarn lengths and the area using actual square units, we will estimate areas using rectangles. Start by drawing a large rectangle that encompasses Rhode Island. Measure the sides and complete the table below. Show your calculations for perimeter and area and be sure to include units.

State	Width	Length	Perimeter	Area
Rhode Island				

4. Work with your partner to use a rectangle to estimate the perimeter and area of Connecticut. Complete the table below and don't forget units.

State	Width	Length	Perimeter	Area
Connecticut				

5. How many rectangles fit inside of Massachusetts? Using a ruler, draw multiple rectangles that make up the area of the state of Massachusetts.



6. Follow along with your teacher to estimate the area and perimeter of Massachusetts. Show all your calculations.

Area Calculations	Perimeter Calculations



Area and Perimeter

Name _____

Date _____

Lesson 6: Assessment

1. Using the map of the United States, find the state of Colorado.

- a. Use a marker to show the perimeter. Mark the color you used here: _____
Describe what the perimeter represents.

- b. Use a marker to show the area. Mark the color you used here
Describe what the area represents.

2. Focus on the entire United States.

- a. Using one-inch yarn lengths, estimate the perimeter of the United States.




- b. Using square-inches, estimate the area of the United States.

- c. Suppose we measured the perimeter in centimeters. Would we get more or less centimeter than when we measured in inches? Why?

- d. Create rectangles and draw them on your map to approximate the area of the United States. Show your work below.

- e. Using your rectangles, find the perimeter of the United States. Make sure you include only "outside" lines in your calculation.

3. Outline the state of Colorado, Wyoming, Utah and Oklahoma. Find the area and perimeter of each using rectangles. Don't forget to use the correct units.

State	Sketch of Rectangles	Perimeter Calculation	Area Calculation
Colorado			
Wyoming			
Utah			
Oklahoma			

			
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